

Running Coupling Corrections to Nonlinear Evolution for Diffractive Dissociation

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Abstract

We determine running coupling corrections to the kernel of the non-linear evolution equation for the cross section of single diffractive dissociation in high energy DIS. The running coupling kernel for diffractive evolution is found to be exactly the same as the kernel of the rcBK evolution equation.

Keywords: diffraction, non-linear evolution, running coupling

1 Introduction

In recent years running coupling corrections have been calculated for a variety of small- x observables. The running coupling corrections for the linear Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation [1, 2] were found in [3] (see also [4, 5]), while the corrections for the non-linear Balitsky-Kovchegov (BK) [6–10] and Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) [11–18] evolution equations were found in [19–22]. The question of the running coupling scale in gluon production was first addressed in [23], with the corrections to the lowest-order gluon production cross section calculated in [24].

Calculations of running coupling corrections for these small- x observables serve two related purposes. One is to answer the theoretical question about how running coupling effects enter small- x evolution and particle production. The other reason for calculating running coupling corrections is phenomenological: it is known that running coupling effects reduce the effective value of the BFKL pomeron intercept [5, 22, 25] as compared to the fixed-coupling results, leading to a slower growth with energy of cross sections and structure functions, which is more in line with the experimental data. This feature resulted in a very successful phenomenology developed over the past few years for the deep inelastic scattering (DIS) [26, 27] and for heavy ion collisions [28] based on the running coupling BK (rcBK) equation.

The existing phenomenological applications of the running coupling evolution to DIS mainly concentrate on the total DIS cross section and the structure functions [26, 27]. A related DIS process is the single diffractive dissociation, in which the proton or nuclear target remains intact in the interaction, generating an adjacent rapidity gap, with the incoming virtual photon splitting into a $q\bar{q}$ pair along with several more gluons and quarks, which later fragment into hadrons with the net invariant mass M_X^2 . The process is illustrated in Fig. 1. At very high energy/small- x the cross section of single diffractive dissociation is described by the non-linear

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evolution equation derived in [29] in the leading- $\ln 1/x$ and large- N_c approximations with fixed coupling. The equation was generalized beyond the large- N_c limit in [30–32]. At the same time, until now the running coupling corrections to the equation of [29] have not been constructed. Similar to the running coupling corrections for other small- x observables, these corrections are needed first of all on the purely theoretical grounds, to determine the scale of the running coupling constant in the kernel of the equation of [29]. Second of all, they may be useful phenomenologically to analyze single diffractive dissociation in the same framework as the total DIS cross sections. (Note though that since the invariant mass M_X^2 of the hadrons produced in the single diffractive dissociation at HERA is not very large, in practical phenomenological applications one often limits the virtual photon breakup to the $q\bar{q}$ pair and at most one gluon [33–35], thus alleviating the need for an evolution equation. We will proceed here expecting that such approximation will require small- x evolution corrections at the future DIS machines.) The goal of this work is to find the running coupling corrections to the evolution equation for diffractive dissociation constructed in [29].

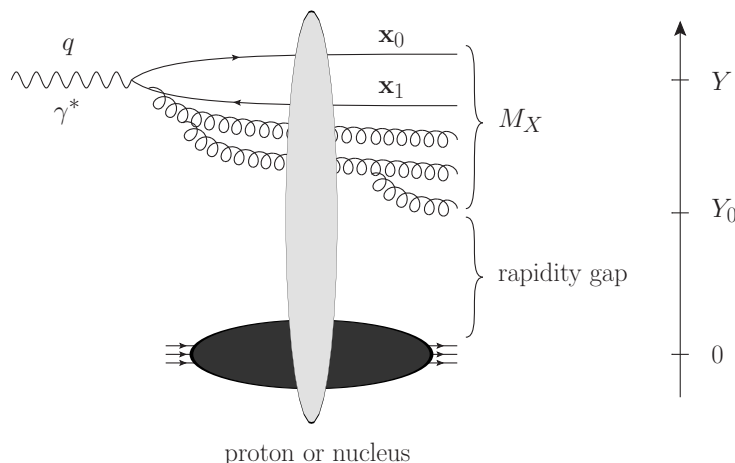


Figure 1: Single diffractive dissociation in high energy DIS.

This letter is structured as follows. In Sec. 2 we re-derive the non-linear evolution for the single diffractive dissociation at fixed-coupling using a method different from the original one used in [29], more similar to the one used in [32]. This new method makes inclusion of the running coupling corrections straightforward, which is accomplished in Sec. 3. Our final result is shown in (21), which, along with the kernels (16) and (18) demonstrates that the running coupling corrections to the kernel of the non-linear evolution equation for diffraction are exactly the same as those for the rcBK and rcJIMWLK evolution equations.

2 Re-deriving the fixed-coupling evolution for single diffractive dissociation

We begin by re-deriving the fixed-coupling evolution equation for the single diffractive dissociation in DIS originally found in [29]. Our goal is to find the cross section $\sigma_{diff}^{\gamma^*A}$ for the single

diffractive dissociation in DIS on a nucleus A resulting in a rapidity gap and production of hadrons with the invariant mass M_X^2 , as shown in Fig. 1. We denote the net rapidity interval between the projectile $q\bar{q}$ pair and the target nucleus by $Y = \ln s/Q^2$, where s is the center-of-mass energy of the $q\bar{q}$ dipole–nucleus system and $Q^2 = -q^2$ is the photon virtuality. As shown in Fig. 1, if we assume that the nucleus has rapidity 0, then the projectile $q\bar{q}$ dipole would have rapidity Y . The rapidity gap stretches from rapidity 0 to rapidity Y_0 . Assuming that $s \gg M_X^2 \gg Q^2$ one has $Y_0 \approx \ln s/M_X^2$.

We are working in the kinematics when both Y_0 and $Y - Y_0$ are very large, such that $\alpha_s Y_0 \sim 1$ and $\alpha_s (Y - Y_0) \sim 1$ and the leading-logarithmic small- x evolution corrections resumming powers of $\alpha_s Y_0$ and $\alpha_s (Y - Y_0)$ are important. That is, we need to resum gluon emissions and virtual corrections for the gluons inside the rapidity gap and in the dissociation region of the $q\bar{q}$ pair leading to creation of hadrons with the invariant mass M_X^2 .

Following [29] we denote by $N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)$ the cross section per unit impact parameter of the single diffractive dissociation in the dipole–nucleus scattering with the rapidity gap greater than or equal to $[0, Y_0]$ and the net rapidity interval of Y . As usual \mathbf{x}_0 and \mathbf{x}_1 are the transverse positions of the quark and the anti-quark in the $q\bar{q}$ dipole, as shown in Fig. 1. (Boldface notation denotes two-component vectors in the transverse plane.) With the help of N^D the differential cross section for the single diffractive dissociation in DIS can be written (for a fixed s) as

$$M_X^2 \frac{d\sigma_{diff}^{\gamma^* A}}{dM_X^2} = - \int d^2x_0 d^2x_1 \int_0^1 dz |\Psi^{\gamma^* \rightarrow q\bar{q}}(x_{01}, z)|^2 \frac{\partial N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)}{\partial Y_0}, \quad (1)$$

with $|\Psi^{\gamma^* \rightarrow q\bar{q}}(x_{01}, z)|^2$ the standard order- α_{EM} light-cone wave function squared for a virtual photon fluctuating into a $q\bar{q}$ pair with $x_{01} = |\mathbf{x}_0 - \mathbf{x}_1|$ (see e.g. [9]).

We want to write down an evolution equation for $N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)$. First we note that when $Y_0 = Y$ the rapidity gap spans the whole rapidity interval and the interaction is elastic. In such case [29]

$$N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y = Y_0, Y_0) = [N_{\mathbf{x}_0, \mathbf{x}_1}(Y_0)]^2 \quad (2)$$

where $N_{\mathbf{x}_0, \mathbf{x}_1}(Y)$ is the dipole–nucleus forward scattering amplitude defined by

$$N_{\mathbf{x}_0, \mathbf{x}_1}(Y) = \frac{1}{N_c} \langle \text{tr} [1 - V_{\mathbf{x}_0} V_{\mathbf{x}_1}^\dagger] \rangle_Y \quad (3)$$

with $V_{\mathbf{x}}$ the Wilson line describing the interaction of a quark at \mathbf{x} moving along the x^+ axis with the target

$$V_{\mathbf{x}} = \text{P exp} \left\{ i g \int_{-\infty}^{\infty} dx^+ t^a A^{a-}(x^+, x^- = 0, \mathbf{x}) \right\}. \quad (4)$$

Here A^μ is the operator of the gluon field of target, t^a are fundamental generators of $\text{SU}(N_c)$, and $\langle \dots \rangle_Y$ in Eq. (3) denotes averaging in the target wave function evolved up to rapidity Y (see [36,37] for reviews of this notation). The light cone coordinates are defined as $x^\pm = (t \pm z)/\sqrt{2}$.

In the large- N_c limit and in the leading- $\ln s$ approximation $N_{\mathbf{x}_0, \mathbf{x}_1}(Y)$ obeys the BK evolution equation [6–10]

$$\partial_Y N_{\mathbf{x}_0, \mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} [N_{\mathbf{x}_0, \mathbf{x}_2}(Y) + N_{\mathbf{x}_2, \mathbf{x}_1}(Y) - N_{\mathbf{x}_0, \mathbf{x}_1}(Y) - N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}(Y)] \quad (5)$$

where $x_{ij} = |\mathbf{x}_{ij}|$ with $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ and $\partial_Y = \partial/\partial Y$. The solution of Eq. (5) would allow us to find the elastic dipole–nucleus scattering cross section (per unit impact parameter) via Eq. (2). The resulting $N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y = Y_0, Y_0)$ would serve as the initial condition for the Y -evolution of $N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)$ we are about to construct.

Note that N^D is a scattering cross section (per unit impact parameter), and, therefore, when written as amplitude squared, contains interactions to the left and to the right of the final state cut. (For example, in Eq. (2) the interactions are contained in the two factors of N .) It is easier to derive its evolution by completing it to the S -matrix like object by defining

$$S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) = 1 - 2 N_{\mathbf{x}_0, \mathbf{x}_1}(Y) + N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0). \quad (6)$$

The new quantity S^D contains the no-interaction term ($= 1$), and two terms with the interactions only to the left or to the right of the cut (giving $-N_{\mathbf{x}_0, \mathbf{x}_1}(Y)$ each), in addition to N^D . The addition of these extra terms which have no new particles produced in the final state to N^D means that all terms in S^D describe the final state with the rapidity gap greater than or equal to Y_0 , allowing for no-interaction contributions to the right and to the left of the cut. The quantity S^D is illustrated in Fig. 2, where the solid vertical straight line denotes the final

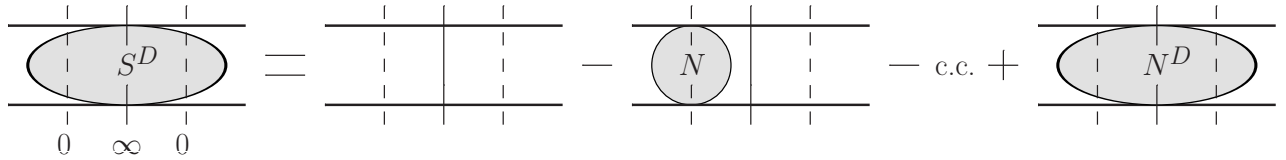


Figure 2: Diagrammatic representation of S^D defined in Eq. (6).

state cut at the light-cone time $x^+ = \infty$, while the dashed vertical straight lines denote the Glauber-Mueller interaction with the target [38] which happens at the time $x^+ = 0$ both in the amplitude and in the complex conjugate amplitude. The times are labeled in the leftmost diagram in Fig. 2. (The time scale of the interaction with the target is much shorter than the time scales of the small- x evolution: see [36, 37, 39] for reviews of similar calculations.) Shaded ovals and the circle in Fig. 2 represent all gluon emissions and interactions with the target nucleus contained in S^D , N , and N^D as labeled.

To construct one step of small- x evolution for S^D we have to see how this quantity changes under an emission of a gluon with rapidity $Y > Y_0$. Note that for $Y = Y_0$ Eq. (2) leads to

$$S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y = Y_0, Y_0) = [1 - N_{\mathbf{x}_0, \mathbf{x}_1}(Y_0)]^2, \quad (7)$$

which is the initial condition for the evolution of S^D . In one step of small- x evolution a soft gluon can be emitted and absorbed by the quark and the anti-quark in the dipole 01 at any time

$x^+ \in (-\infty, +\infty)$ both in the amplitude and in the complex conjugate amplitude. The rapidity gap constraint on the final state of S^D implies that no gluon should be present at $x^+ = \infty$ for gluon rapidities less than Y_0 . However, this condition is already satisfied by our initial condition (7). There are no final-state restrictions on the gluons with $Y > Y_0$: they may or may not go through the cut. This makes the gluon emissions and absorptions at times $x^+ \in (0, +\infty)$ in the amplitude and in the complex conjugate amplitude cancel, due to the final-state cancellations originally derived in [40] (see e.g. [29, 41] for applications of such cancellations). The final state cancellations of [40] are illustrated in Fig. 3 which shows how the diagrams with the gluon either emitted or absorbed (or both) at $x^+ \in (0, +\infty)$ cancel. (Cancellations also take place for all other couplings of the gluon to the dipole not shown explicitly in Fig. 3.)

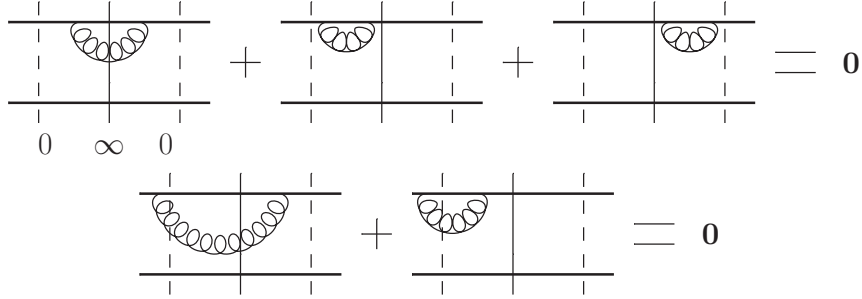


Figure 3: Diagrammatic examples of the final state cancellations derived in [40]. Cancellation for the sum of the diagrams which are complex conjugates of those shown in the bottom panel also takes place, but is not shown explicitly.

This leaves us with gluon emissions/absorptions at $x^+ \in (-\infty, 0)$ both in the amplitude and in the complex conjugate amplitude. Identifying the time interval $x^+ \in (-\infty, 0)$ in the complex conjugate amplitude with the time interval $x^+ \in (0, +\infty)$ in the forward amplitude we conclude that the evolution for S^D with $Y > Y_0$ is the same as for the S -matrix of the dipole–nucleus scattering, which is related to the forward amplitude N by

$$S_{\mathbf{x}_0, \mathbf{x}_1}(Y) = 1 - N_{\mathbf{x}_0, \mathbf{x}_1}(Y). \quad (8)$$

In the large- N_c limit it is described by the BK equation (5) re-written for the S -matrix:

$$\partial_Y S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2} [S_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) S_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)]. \quad (9)$$

The evolution equation (9) is illustrated in Fig. 4, where, in the large- N_c approximation notation gluons are denoted by the double lines and disconnected gluon (double) line implies all possible connections to the quark and the anti-quark in the dipole (see [42–44]).

One could stop at Eq. (9), since its solution with the initial condition (7) used in Eq. (6) would give one N_D . (Even more, Eq. (6) implies that $\partial_{Y_0} S^D = \partial_{Y_0} N^D$, which is all that is needed to find the differential diffractive cross section in Eq. (1).) Alternatively one can use

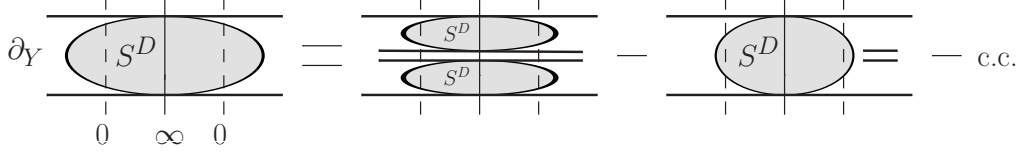


Figure 4: The evolution equation for S^D .

Eq. (6) along with Eq. (5) in Eq. (9) to obtain the evolution equation for N^D [29]

$$\begin{aligned} \partial_Y N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) = & \frac{\alpha_s N_c}{2\pi^2} \int d^2 x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2} \left[N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) + N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) \right. \\ & + N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - 2 N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - 2 N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) N_{\mathbf{x}_2, \mathbf{x}_1}(Y) \\ & \left. + 2 N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}(Y) \right]. \quad (10) \end{aligned}$$

3 Running coupling corrections

Similar to the previous works on the running coupling corrections to small- x observables [19–24] we use the Brodsky–Lepage–Mackenzie (BLM) prescription [45]: one has to dress all the gluon propagators and multi-gluon vertices with quark bubble corrections, and then replace the number of quark flavors N_f by

$$N_f \rightarrow -6\pi\beta_2 \quad (11)$$

where

$$\beta_2 = \frac{11N_c - 2N_f}{12\pi} \quad (12)$$

is the one-loop QCD beta-function. In the end one is able to absorb all such corrections into the powers of the physical running couplings

$$\alpha_s(Q^2) = \frac{\alpha_\mu}{1 + \alpha_\mu \beta_2 \ln \frac{Q^2}{\mu^2}}. \quad (13)$$

This procedure fixes the scales Q in the running couplings.

When the gluon lines in our above derivation of Eq. (10) are dressed by the quark loop corrections, the final state cancellations of [40] shown in Fig. 3 still take place. All the $g \rightarrow q\bar{q}$ splitting and the $q\bar{q} \rightarrow g$ mergers happening at the times $x^+ \in (0, +\infty)$ in the amplitude and in the complex conjugate amplitude cancel (simultaneously with the cancellation of gluon emissions and absorptions by the original dipole at $x^+ \in (0, +\infty)$ shown in Fig. 3). These cancellations have been discussed in detail in [23]. An example of a cancellation of the final state interactions with a quark loop correction added to the top line of Fig. 3 is shown in Fig. 5: as we demonstrate in Appendix A, the sum of all the diagrams shown in Fig. 5 is zero,

$$A + B + C + D + E = 0. \quad (14)$$

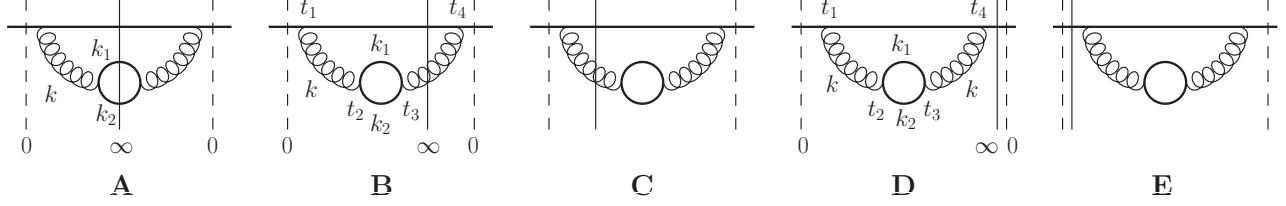


Figure 5: An example of the final-state interactions with a quark loop correction: the sum of these diagrams is zero, as shown in Appendix A.

The argument can be repeated at higher orders, showing that higher numbers of quark loops would also cancel in the final state, and for other diagrams with the final-state interactions.

The remaining $g \rightarrow q\bar{q}$ splitting and $q\bar{q} \rightarrow g$ mergers are limited to the $x^+ \in (-\infty, 0)$ times both in the amplitude and in the complex conjugate amplitude, thus dressing the propagators of the early-time gluons only. Therefore, the running coupling evolution for S^D is the same as the running coupling evolution for the BK equation (5) written for the dipole–nucleus S -matrix of Eq. (8). Using the results of the derivation of the rcBK evolution in [19, 20] we write the running coupling evolution equation for S^D as

$$\partial_Y S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) = \int d^2 x_2 K(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) [S_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) S_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - S_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0)]. \quad (15)$$

The kernel $K(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$ of the evolution equation (15) depends on the prescription used to fix the scales of the running couplings (see [22] for details). In the Balitsky prescription [20] the kernel is

$$K_{rc}^{Bal}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) = \frac{N_c \alpha_s(x_{10}^2)}{2\pi^2} \left[\frac{x_{10}^2}{x_{20}^2 x_{21}^2} + \frac{1}{x_{20}^2} \left(\frac{\alpha_s(x_{20}^2)}{\alpha_s(x_{21}^2)} - 1 \right) + \frac{1}{x_{21}^2} \left(\frac{\alpha_s(x_{21}^2)}{\alpha_s(x_{20}^2)} - 1 \right) \right], \quad (16)$$

with the following shorthand notation

$$\alpha_s(x_\perp^2) = \alpha_s \left(\frac{4 e^{-\frac{5}{3} - 2\gamma_E}}{x_\perp^2} \right) \quad (17)$$

in the $\overline{\text{MS}}$ renormalization scheme. In the Kovchegov–Weigert prescription [19] the kernel is

$$K_{rc}^{KW}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) = \frac{N_c}{2\pi^2} \left[\alpha_s(x_{20}^2) \frac{1}{x_{20}^2} - 2 \frac{\alpha_s(x_{20}^2) \alpha_s(x_{21}^2)}{\alpha_s(R^2)} \frac{\mathbf{x}_{20} \cdot \mathbf{x}_{21}}{x_{20}^2 x_{21}^2} + \alpha_s(x_{21}^2) \frac{1}{x_{21}^2} \right] \quad (18)$$

with

$$R^2 = x_{20} x_{21} \left(\frac{x_{21}}{x_{20}} \right)^{\frac{x_{20}^2 + x_{21}^2}{x_{20}^2 - x_{21}^2} - 2} \frac{x_{20}^2 x_{21}^2}{\mathbf{x}_{20} \cdot \mathbf{x}_{21}} \frac{1}{x_{20}^2 - x_{21}^2}. \quad (19)$$

The terms in the kernel neglected by each prescription can be found in [20, 22].

The initial condition for the running coupling evolution for S^D is given by Eq. (7), where N is now found from the rcBK equation [19, 20]

$$\partial_Y N_{\mathbf{x}_0, \mathbf{x}_1}(Y) = \int d^2 x_2 K(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) [N_{\mathbf{x}_0, \mathbf{x}_2}(Y) + N_{\mathbf{x}_2, \mathbf{x}_1}(Y) - N_{\mathbf{x}_0, \mathbf{x}_1}(Y) - N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}(Y)]. \quad (20)$$

Finally, using Eq. (6) in Eq. (15) one obtains the running coupling evolution for N^D

$$\begin{aligned} \partial_Y N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) = \int d^2 x_2 K(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) & \left[N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) + N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - N_{\mathbf{x}_0, \mathbf{x}_1}^D(Y, Y_0) \right. \\ & + N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - 2 N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}^D(Y, Y_0) - 2 N_{\mathbf{x}_0, \mathbf{x}_2}^D(Y, Y_0) N_{\mathbf{x}_2, \mathbf{x}_1}(Y) \\ & \left. + 2 N_{\mathbf{x}_0, \mathbf{x}_2}(Y) N_{\mathbf{x}_2, \mathbf{x}_1}(Y) \right] \quad (21) \end{aligned}$$

with the kernel given by Eq. (16) or by Eq. (18) depending on the prescription choice and with the initial condition given by Eq. (2) with N provided by Eq. (20).

Eqs. (15) and (21) are the main results of this letter, providing running coupling corrections to the non-linear small- x evolution for the cross section of diffractive dissociation in DIS. The kernels in Eqs. (16) and (18) have of course been found before in the calculation of the rcBK and rcJIMWLK evolutions [19, 20]: we showed that these same kernels govern the evolution for diffractive dissociation. Our derivation was done in the large- N_c limit: since the evolution equation (15) is simply the rcBK equation, its generalization to all- N_c is the same as for the rcBK equation, and is accomplished by performing the $\langle \dots \rangle_{Y, Y_0}$ averaging to the whole right-hand side of Eq. (15) instead of applying it to each individual S^D separately (see [15, 16]). Note that the averaging should now include the final-state rapidity gap greater than or equal to $[0, Y_0]$.

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A Final state cancellations

We want to demonstrate the cancellation of diagrams in Fig. 5. We will be working in the light-cone perturbation theory (LCPT) [46, 47]. In LCPT the diagrams in Fig. 5 differ only by their energy denominators and overall signs. Suppressing the rest of the diagrams we write for the contribution of the diagram A

$$A = \frac{1}{(E_1 + E_2)^2 (E_1 + E_2 - E_k)^2} \quad (A1)$$

where $E_k = \mathbf{k}^2/(2k^+)$ is the light cone energy of a line with momentum k , and $E_1 = \mathbf{k}_1^2/(2k_1^+)$, $E_2 = \mathbf{k}_2^2/(2k_2^+)$ (momentum labels are explained in Fig. 5).

The remaining diagrams have to be treated more carefully, due to the presence of intermediate states with zero light-cone energy denominators [23, 40]. Noting that the diagram B has an overall minus sign compared to A due to moving a quark-gluon vertex across the cut we write

$$B = - \int_0^\infty dt_3 e^{i(E_k - E_1 - E_2 + i\epsilon)t_3} \int_0^{t_3} dt_2 e^{i(E_1 + E_2 - E_k + i\epsilon)t_2} \int_0^{t_2} dt_1 e^{i(E_k + i\epsilon)t_1} \int_0^\infty dt_4 e^{-i(E_k - i\epsilon)t_4} \\ = \frac{i}{2\epsilon E_k^2 (E_1 + E_2 - E_k)} + \frac{2E_1 + 2E_2 - 3E_k}{2E_k^3 (E_1 + E_2 - E_k)^2} + O(\epsilon), \quad (\text{A2})$$

where all the light cone times associated with the vertices, $t_i = x_i^+$, are labeled in Fig. 5B and ϵ is an infinitesimal regulator. (See [23, 40] for similar calculations.) One can show that the singular $O(1/\epsilon)$ term in Eq. (A2) is canceled by the diagrams with the gluon lines replaced by the instantaneous interactions of LCPT: we thus concentrate on the $O(\epsilon^0)$ term. Also, due to mirror symmetry, $C = B^*$, such that $O(1/\epsilon)$ terms would cancel in the $B + C$ sum [40].

Similarly for the diagrams D and E we get (see Fig. 5D for notations)

$$D = \int_0^\infty dt_4 e^{i(-E_k + i\epsilon)t_4} \int_0^{t_4} dt_3 e^{i(E_k - E_1 - E_2 + i\epsilon)t_3} \int_0^{t_3} dt_2 e^{i(E_1 + E_2 - E_k + i\epsilon)t_2} \int_0^{t_2} dt_1 e^{i(E_k + i\epsilon)t_1} \\ = \frac{i}{4\epsilon E_k^2 (E_1 + E_2)} - \frac{2E_1 + 2E_2 + E_k}{2E_k^3 (E_1 + E_2)^2} + O(\epsilon). \quad (\text{A3})$$

Again, due to mirror symmetry, $E = D^*$. With the help of Eqs. (A1), (A2), and (A3) and dropping the $O(\epsilon)$ terms we see that

$$A + B + C + D + E = 0, \quad (\text{A4})$$

as desired.

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